Relations Paper I 13/04/21

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Relation: – let A and B be two sets. A relation R from A to B is defined as a subset of $A \times A$

i.e. $R \subseteq A \times B$.

If $(a,b) \in R$, then we say that a is R related to b)

Symbolically $R = \{(a,b) : a \in A, b \in B \text{ and } aRb)$

i.e. if $A = \{2,3,4,5\}$, $B = \{2,4,6\}$ then the relation R from A to B given by aRb if $a < b$

$A \times B = \{(2,2), (2,4), (2,6), (3,2), (3,4), (3,6), (4,2), (4,4), (4,6), (5,2), (5,4), (5,6)\}$.

for $a < b$ we have

$R = \{(2,4), (2,6), (3,4), (3,6), (4,6), (5,6)\}$.

Equivalence Relation: – A relation R defined on a set A is an equivalence relation iff it satisfies all the following three conditions.

(i) R is reflexive
i.e; $aRa \ \forall a \in A$

(ii) R is symmetric
i.e, $aRb \Rightarrow bRa \ \forall a,b \in A$

(iii) R is transitive
i.e, aRb and $bRc \Rightarrow aRc \ \forall a,b,c \in A$.

Theorem :- Show that inverse of an equivalence relation is also an equivalence relation.

Proof :- Given that R is an equivalence relation then in set X then R must be reflexive, symmetric and transitive

let $a, b, c \in X$ then we have for R^{-1}

(i) Reflexive :- $(a,a) \in R^{-1}$, for $(a,a) \in R \; \forall \; a \in X$

$\Rightarrow (a,a) \in R^{-1}$

i.e. R^{-1} is reflexive.

(ii) symmetric :- $(a,b) \in R^{-1} \Rightarrow (b,a) \in R^{-1}$

For $(a,b) \in R^{-1} \Rightarrow (b,a) \in R$

$\Rightarrow (a,b) \in R$ $\quad$ ($\because R$ is symmetric)

$\Rightarrow (b,a) \in R^{-1}$

$\therefore R^{-1}$ is symmetric.

Also $(a,b) \in R^{-1} \Rightarrow (a,b) \in R \quad \therefore R^{-1} = R$ in this case.

(iii) Transitive :- $(a,b), (b,a) \in R^{-1}$

$\Rightarrow (a,c) \in R^{-1}$

We have, $(a,b), (b,c) \in R^{-1}$

$\Rightarrow (b,a) \; (c,b) \in R$

$\Rightarrow (c,b) \; (b,a) \in R$

$\Rightarrow (c,a) \in R \quad \Rightarrow (a,c) \in R^{-1}$

$\therefore R^{-1}$ is transitive.

$\therefore R^{-1}$ is an equivalence relation in X.